Lattice Boltzmann simulation of heat transfer with phase change in saturated soil during freezing process

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ABSTRACT
A lattice Boltzmann model is presented for simulating heat transfer with phase change in saturated soil. The model includes a quartet structure generation set for creating soil structure, double distribution functions for simulating temperature field evolution of soil particles and water, respectively, and an enthalpy-based method for tracing phase interface. The model is validated by two cases with analytical solutions. Then, we investigate the influence of porosity on freezing process in saturated sandy loam soil. The results demonstrate that porosity is the predominant factor when the location is far from the cold source; otherwise, thermal gradient is more important.

1. Introduction
Heat transfer with phase change in saturated soil is of practical importance to many engineering problems, such as natural freezing of soil in cold regions, artificial freezing of ground during subway construction, soil freezing around ground heat exchangers, etc [1–6]. If engineers or researchers cannot clearly grasp the mechanism of temperature field evolution during soil freezing, it will seriously affect the safety of project and may cause heavy lives and economic losses. One of the most serious accidents related to soil freezing happened in Shanghai Metro Line 4 on July 1, 2003. Due to the collapse of freezing wall of cross passage, mixture of water and sand from Huangpu River poured into the twin tunnels, the adjacent buildings were adversely destroyed and the completed tunnels were damaged as a result, which produced economic losses nearly 20 million dollars [7]. Therefore, a better understanding of heat transfer mechanism in soil is essentially important, and special attention should be paid to the problem during freezing process.

Numerous experimental and numerical studies have been done and many classic models have been developed on the issue of soil freezing process. Most works are based on the macroscopic continuum method, which regard the multiphase soil material as homogeneously dispersed system ignoring the interaction among different components [3, 8–10]. In fact, soil is a kind of granular porous media, and lots of studies indicate that heat transfer in porous media is strongly affected by its pore structure [11–13]. But traditional continuum theories can hardly describe the complex pore boundaries. And few experiments can get the accurate results of heat transfer at microscopic scale referring to the interaction between fluid and solid. It is therefore desirable to look for alternative method no longer based on continuum assumptions, but able instead to capture the mesoscopic nature of heat transfer with phase change in soil during freezing process.

The lattice Boltzmann method (LBM), as a mesoscopic numerical method, is based on the kinetic equations of particle distribution functions. In recent years, the LBM has been successfully applied to simulate the pore-scale heat transfer in porous media, owing to its numerical stability,
inherently parallelizability, simple implementation, and ability to handle complex geometry and boundary conditions. To solve the temperature evolution equation coupling with the nonlinear latent heat source term, Jiaung [14] first developed a lattice Boltzmann model for simulating phase change governed by the heat conduction equation incorporated with enthalpy formation. Subsequently, Chatterjee [15], Semma [16], Huber [17], Semma [18], Eshraghi [19], et al published a series of papers on simulating solid–liquid phase change problem using LBM with enthalpy approach to treat latent heat effect on the temperature field. The above studies are mainly aimed at heat transfer problem of pure substance. For porous media, simulation at both pore scale and representative elementary volume (REV) scale can be conducted with LBM. Guo [20] extended the LBM to model the convection heat transfer in porous media at the REV scale. Gao [21] proposed an enthalpy-based LBM model to investigate natural convection with solid–liquid phase change in porous media at the REV scale. Liu [22] developed a double MRT-LB model for simulating transient solid–liquid phase change problems in porous media at the REV scale. Wang [23] presented a LBM model to simulate fluid–solid conjugate heat transfer by implementing an interface boundary. Wang [24] studied the effective thermal conductivity of two-phase porous media for a certain porosity based on the pore scale. Song [6] studied the heat and mass transfer phenomenon with phase transformation in soil during freezing process. Almost all the works relevant are either at the REV scale neglecting the influence of pore structure or at the pore scale without considering the phase change problem for simplifying the calculation.

The aim of this work is to study the heat transfer with phase change in saturated soil during freezing process with LBM. For this purpose, we use the quartet structure generation set (QSGS) to create the stochastic pore structure of soil, and then based on the LBM with two-dimensional four-speed (D2Q4) model, the double distribution functions are applied to simulate temperature field evolution, one is for soil particles, and the other is for water. To reflect the phase change process during water freezing into ice, the enthalpy approach is used to trace the solid–liquid interface by updating the liquid-phase fraction. The model is subsequently tested to simulate solid–liquid phase change of pure substance and fluid–solid conjugate heat transfer of dual-component materials with two basic structures: series mode and parallel mode. At last, we discussed the temperature field evolution of the saturated sandy loam soil with different porosities during freezing process.

<table>
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2. Reconstruction of soil microstructure

In nature, soil is a heterogeneous assemblage of materials, forming a porous structure. In general, it is difficult to accurately describe the microstructure of soil due to its complexity and randomicity. To simulate the heat transfer problem with phase change in saturated soil at the pore scale, it is necessary to acquire the microstructure and the distribution information of different phases [23].

2.1. Description of soil microstructure

The saturated soil consists of two parts: the pore (or water) and the solid particles. The distribution of particles and pore is disordered, so the binary random function $Z(x)$ is usually used to model the soil microstructure, $Z(x)$ is defined as follows:

$$Z(x) = \begin{cases} 1 & \text{if } x \text{ is pore} \\ 0 & \text{otherwise} \end{cases}$$

It is obvious that the mean of $Z(x)$ is the porosity $\varepsilon$ of soil.

2.2. Soil microstructure generation

Several methods have been put forward to reconstruct the microstructure of porous media in the past decades: multiple-point geostatistics [25], particle packing model [26], network generation model [27], fractal model [28], the QSGS method [29–31], etc. Among all of these, the QSGS method, proposed by Wang in 2007 [29], has been widely used in the reconstruction of porous media due to its clear physical meaning, capability of considering the solid particles’ random distribution, and statistical characteristics. In this paper, the QSGS method is applied for soil microstructure generation according to its porosity. Assuming the solid particles are the growth phase, and the whole lattice nodes of construction area are all pores at the beginning. The generation process follows the steps below:

i. Assign $p_c$ as the initial distribution probability to generate the solid-phase core randomly, and promise that $p_c$ is less than the volume fraction of solid phase (i.e., $p_c < 1 - \varepsilon$).

ii. Take solid-phase core as growing center and apply the growth probability $p_i$ in different directions to make solid-phase core grow on the neighboring lattice nodes. The subscript $i$ is the direction of growth, $i = 1, 2, 3 \ldots 8$, which is shown in Figure 1.

iii. Repeat the growing process of (ii) until volume fraction of pores reaches the given porosity $\varepsilon$.

For three phases of porous media, the abovementioned steps can be applied to generate each phase ignoring the interaction between these phases. Otherwise, the probability $p_i^{nm}$ should be introduced to characterize the growth probability of the $n$th phase on the $m$th phase in the $i$th direction. Therefore, the QSGS method can control the microstructure of porous media by setting the four parameters ($p_c$, $p_n$, $p_i^{nm}$, and $\varepsilon$). For the two-dimensional saturated soil in this paper, the microstructure is

![Figure 1. Solid-phase core growth direction. The central black square is the growing core of solid. The numbers 1–8 represent the 8 growth directions of solid core.](image-url)
produced by setting the parameters \((p_c, p_i, \text{ and } \varepsilon)\), without considering the interaction between both phases. Figure 2 is an example of reconstructed soil with the QSGS method.

3. Heat transfer model of saturated soil during freezing process

Based on the assumptions that soil is saturated, and composed of two parts: soil particles (solid phase) and water (liquid phase), the energy equations for heat transfer with phase change during freezing process in the multiphase system can be written, respectively, as:

Energy equation of solid phase

\[
(\rho C_p)_s \frac{\partial T}{\partial t} = k_s \nabla^2 T
\]  

Energy equation of liquid phase with phase change

\[
(\rho C_p)_f \frac{\partial T}{\partial t} = k_f \nabla^2 T + \rho_f \left( \frac{\partial \Delta H}{\partial t} \right)
\]  

Figure 2. The microstructure of reconstructed soil with QSGS method. White for pore and black for solid. Note: QSGS, quartet structure generation set.

where subscript \(f\) represents the fluid, and \(s\) the solid; \(T\) is the temperature. \(\rho\), \(C_p\), \(k\) are density, specific heat, and thermal conductivity, respectively; \(t\) is the time; \(\Delta H\) is the amount of heat released due to phase change.

To simplify the numerical calculations, the energy equation with phase change for liquid phase is generally expressed in terms of total enthalpy, thus the governing Equation, Eq. (3), can be written as

\[
\rho_f \left( \frac{\partial H}{\partial t} \right) = k_f \nabla^2 T
\]  

the total enthalpy \(H\) has two parts, namely, the sensible enthalpy and latent heat enthalpy, so it can be written as

\[
H = C_p T + f_l L_a
\]  

where \(f_l\) is the liquid phase fraction, and is zero for the solid region, 1 for the liquid region. \(L_a\) is the latent heat of phase change.
Substituting Eq. (5) into Eq. (4) yields
\[
\frac{\partial T}{\partial t} = \alpha_f \nabla^2 T + Sr
\]
where \( \alpha_f \) is the thermal diffusivity, \( \alpha_f = k_f / (\rho C_p)_f \); \( Sr \) is the heat source term \( Sr = -\left( \partial f_i / \partial t \right) L_a / (C_p)_l \).

When the phase change temperature is \( T_f \), the total enthalpy of solid region \( H_s \) equals to \( C_p T_f \), and the liquid region \( H_l \) is \( C_p T_f + L_a \). So the relationship between \( H \) and \( T \) can be established as
\[
T = \begin{cases} \frac{H}{C_p} & H < H_s \\ \frac{T_f}{T_f} & H_s \leq H \leq H_l \\ \frac{H - L_a}{C_p} & H > H_l \end{cases}
\]

Once the total enthalpy \( H \) is determined, the liquid phase fraction \( f_l \) can be expressed as
\[
f_l = \frac{H - H_s}{H_l - H_s}
\]

Thus, Eq. (2) can be used to simulate heat transfer of the solid phase in the soil, and Eq. (6)–Eq. (8) could describe heat transfer problem with phase change for the liquid phase.

To have a clear understanding of physical problem and to simplify the number of parameters, the dimensionless variables are introduced as
\[
\begin{align*}
\xi^* &= \frac{x}{L} \\
t^* &= \frac{a t}{L^2} \\
T^* &= \frac{T - T_0}{T_i - T_0} \\
Ste &= \frac{C_p(T_f - T_0)}{L_a}
\end{align*}
\]

where \( \xi^*, t^*, T^* \) are the dimensionless coordinate, time, and temperature, respectively. \( Ste \) is the Stefan number. \( L \) is the characteristic length. \( T_0 \) is the temperature of cold source. \( T_i \) is the initial temperature of soil.

### 4. Lattice Boltzmann model

The LBM is a mesoscopic approach based on the evolution of statistical particle distribution of a lattice gas whose density represents the physical quantities to be modeled, such as temperature. Its main idea [32–33] is to establish the relationship between microscale and macroscale by not considering each particle behavior alone but behavior of a collection of particles as a unit. This method is effective to study the macroscopic relation and the microscopic mechanism of energy transport process in porous media, and it has been extensively adopted for investigating the transport problems at the pore scale.

#### 4.1. Lattice Boltzmann equation

The D2Q4 LB model (Figure 3) for single relaxation time (or LBGK), which was proposed by Qian [34], is used to solve the heat transfer problem in the saturated soil, which is governed by Eq. (2) and Eq. (3). In the present study, we choose the double distribution functions to model the temperature field evolution in the saturated soil at the pore scale: one is for soil particles (solid phase), the other is for water (liquid phase).

#### 4.1.1. Evolution equation for heat transfer of solid phase

For the solid phase without phase change, the temperature evolution equation is generally written as
\[
g_{si}(r + e_i \delta_t, t + \delta_t) - g_{si}(r, t) = -\frac{g_{si}(r, t) - g_{si}^q(r, t)}{\tau_s} \quad (i = 0, 1, 2, 3)
\]

where \( g_{si} \) represents the distribution function of solid particles, \( \tau_s \) is the relaxation time for solid particles.
where $\tau$ is the dimensionless relaxation time, its value should insure to be within $(0.5, 2)$ \cite{23, 24};

$g_i(r, t)$ is the temperature distribution function of the $i$th direction at the lattice site $r$ and time $t$; $g_i^{eq}(r, t)$ represents the equilibrium distribution function; $e_i$ is the discrete velocity in the lattice

$$e_i = c[(1, 0), (-1, 0), (0, 1), (0, -1)]$$  \hspace{1cm} (11)

where $c$ is the lattice speed, and $c = \delta_x/\delta_t$, $\delta_x$, $\delta_t$ are the lattice space and the time step, respectively. The equilibrium distribution function $g_i^{eq}(r, t)$ can be described as

$$g_i^{eq}(r, t) = \omega_i T(r, t)$$  \hspace{1cm} (12)

where $\omega_i$ represents a weight factor along the $i$th direction.

$$\omega_0 = \omega_1 = \omega_2 = \omega_3 = 1/4$$  \hspace{1cm} (13)

The equilibrium distribution function $g_i^{eq}(r, t)$ satisfies the following relations

$$\sum_{i=0}^{3} g_i^{eq}(r, t) = \sum_{i=0}^{3} \omega_i T(r, t) = T(r, t)$$  \hspace{1cm} (14)

The macroscopic temperature can be calculated as

$$\sum_{i=0}^{3} g_i(r, t) = T(r, t)$$  \hspace{1cm} (15)

\subsection*{4.1.2. Evolution equation for heat transfer of liquid phase with phase change}

For the water in soil, liquid phase is gradually turned into solid phase with the decreasing temperature. To simulate the phase change process of water in the soil, the heat source term $Sr$ of Eq. (6) needs to be retained in the discretized Boltzmann equation, thus the discretized phase change LBM takes the form

$$g_{\beta}(r + e_i \delta_t, t + \delta_t) - g_{\beta}(r, t) = - \frac{g_{\beta}(r, t) - g_{\beta}^{eq}(r, t)}{\tau_f} + \omega_i Sr \delta_t \hspace{1cm} (i = 0, 1, 2, 3)$$  \hspace{1cm} (16)

For simplifying the calculation, the heat source term $Sr$ is discretized into

$$Sr = - \frac{La}{C_p} \frac{[f_i(t + \delta_t) - f_i(t)]}{\delta_t}$$  \hspace{1cm} (17)
Applying Eq. (17) to the right side of Eq. (16), we can rewrite Eq. (16) as

\[ g_f(r + e_i \delta_i, t + \delta_i) - g_f(r, t) = \frac{g_f(r, t) - g_f^{eq}(r, t)}{\tau_f} - \alpha_i \frac{L^a}{C_p} [f_i(t + \delta_i) - f_i(t)] \]

\((i = 0, 1, 2, 3)\)

By Chapman–Enskog expansion, Eq. (18) recovers the macroscopic energy equations, Eq. (3), and the thermal diffusivity \(\alpha\) is given by

\[ \alpha = c_i^2 \left( \tau_f - \frac{1}{2} \right) \delta_i \]

(19)

where \(c_i\) is the lattice sound speed, \(c_i = c^2/2\).

In the process of soil freezing, the liquid water is gradually solidified into ice, and the thermal diffusivity then changes with it. Considering the variation of dimensionless relaxation time \(\tau\) with liquid phase fraction \(f_i\), it can be written as

\[ \tau_f = \frac{f_i \gamma_i + (1 - f_i) \gamma_{ls}}{\delta_i c_i^2} + \frac{1}{2} \]

(20)

where subscript \(ll\) represents the water in the liquid state, and \(ls\) in the solid state.

### 4.2. Boundary conditions

The accurate modeling of boundary conditions is one of the important and crucial issues in the LBM. Usually, it is somehow straightforward to specify boundary condition of energy equations for heat transfer. However, this is not the case for LBM, as the unknown temperature distribution functions at the boundary, lattice nodes must be obtained through the macroscopic boundary conditions. In the present work, the nonequilibrium extrapolation approach, which was proposed by Guo [35] in 2002, is used due to its simplicity, second-order accuracy, and good robustness. The main idea of this approach is to divide the temperature distribution functions at the boundary lattice nodes into two parts, equilibrium part and nonequilibrium part, which can be written as

\[ g_i(B, t) = g_i^{eq}(B, t) + g_i^{neq}(B, t) \]

(21)

The equilibrium part \(g_i(B, t)\) can be got by Eq. (12) according to the temperature boundary conditions. The nonequilibrium part \(g_i^{neq}(B, t)\) is approximated by extrapolating from the neighboring lattice node, it can be expressed as

\[ g_i^{neq}(B, t) = g_i(O, t) - g_i^{eq}(O, t) \]

(22)

### 4.3. Normalizing and scaling

In the process of numerical calculation with LBM, the lattice unit is used in the program parameters, but the physical units are usually adopt in the actual physical problems. Therefore, it is necessary to establish the corresponding relations between them. In general, it is more convenient to solve the specific problem in dimensionless form rather than dimensional one. Under the premise of ensuring the consistency of heat transfer criterion, the dimensionless parameters can be used as a bridge to realize the conversion between two unit systems.

The relation between the physical time \(t\) and lattice time step \(N\) can be established based on the dimensionless time

\[ t^* = \frac{\alpha_p t_p}{L_p^2} = \frac{\alpha_L t_L}{L_L^2} \]

(23)

where \(\alpha_p\) and \(\alpha_L\) are the thermal diffusivity in physical and lattice unit, respectively; \(t_p\) and \(t_L\) are the time in physical and lattice unit, and \(t_L = N \delta_i\), \(N\) is the number of time step; \(L_p\) and \(L_L\) are the
reference length in physical and lattice unit, if $L_p$ is the length of the whole calculation domain, then $L_L = n \delta_x$, $n$ is the number of lattices in length direction.

According to the dimensionless temperature, the relationship between the physical unit and lattice unit for the phase change parameters (the latent heat of phase change $L_a$, specific heat $C_p$) can be established based on the dimensionless Stefan number $Ste$

$$Ste = \frac{C_{pp}(T_{fp} - T_{op})}{L_{ap}} = \frac{C_{pt}(T_{fl} - T_{ol})}{L_{al}}$$  \hspace{1cm} (24)

where subscript $p$ represents the physical unit and $L$ the lattice unit.

In the similar way, we can obtain the corresponding relationship for the length, temperature, and thermal diffusivity of different materials by dimensionless variables.

### 4.4. Numerical procedure

The evolution law for temperature distribution functions of solid phase and liquid phase are governed by Eq. (10) and Eq. (18), respectively. To facilitate the programming, the calculation process is usually divided into two steps: collision and streaming. As an example, the liquid-phase evolution process can be expressed as

**Collision step**

$$g_l(r, t + \delta t) - g_l(r, t) = -\frac{g_l(r, t) - g_{le}(r, t)}{\tau_l} + \omega_s \delta t$$  \hspace{1cm} (25)

**Streaming step**

$$g_l(r + \varepsilon_i \delta r, t + \delta t) = g_l(r, t + \delta t)$$  \hspace{1cm} (26)

Considering the latent heat effects, solid–liquid interface movement, and the thermal diffusivity variation, the LB model has been established for the saturated soil during freezing process. The main calculation steps are as follows:

i. Create the soil microstructure with solid particles and pores through QSGS method

ii. Make unit conversion between the physical and LB models based on the dimensionless parameters.

iii. Initialize the parameters: temperature distribution function $g_l(r, t)$, temperature $T$, liquid-phase fraction $f_l$, etc.

iv. Calculate the total enthalpy $H$ and liquid-phase fraction $f_l$ to judge the state of water in the soil.

v. Execute the collision operation of water (or ice) and solid soil particles, respectively, by setting different dimensionless relaxation time $\tau$.

vi. Execute the streaming operation through the whole computational area, including water (or ice) and soil particles.

vii. Put the boundary conditions into effect: the constant temperature boundary and adiabatic boundary are related in this paper.

viii. Update the temperature distribution functions to get the macroscopic temperature for the next time step.

ix. Make sure the time step reaches the predetermined one, and then output the results; otherwise, repeat the points (iv) ~ (viii).

### 4.5. Verification of LBM model

**Case 1:** Solid–liquid phase change of pure substance

In this case, a solidification problem is simulated with $400 \times 10$ uniform lattice system. The parameters are normalized subsequently, and the thermophysical properties of the pure substance are set as the uniform initial temperature $T_i = 1.0$, boundary temperature of cold source $T_o = 0.0$, etc.
freezing temperature $T_f = 0.5$, latent heat of phase change $L_a = 0.5$, specific heat $C_p = 1.0$, thermal diffusivity of solid phase $\alpha_s = 0.25$. Other parameters are set as follows: lattice spaces are equal in horizontal and vertical direction, $\delta_x = \delta_y = 1$. The time step $\delta t = 1$. The location of liquid-phase fraction $f_l = 0.5$ is defined as the solid–liquid interface.

Figure 4 shows the variety trend of solid–liquid interface with time when $\alpha_s/\alpha_l$ equals 1, 2, and 5, respectively. It can be seen that the LBM results are in good agreement with the analytical ones. At the time step of 10000 ($t^* = 0.015625$), the relative error of LBM results deviated from the analytical results is 0.47, 0.16, 0.61%, respectively, when $\alpha_s/\alpha_l = 1$, 2, and 5. Figure 5 shows the temperature distribution as a function of position at the time step of 10000 ($t^* = 0.015625$). The slope change is observed due to the changes of $\alpha_s/\alpha_l$, and the decreasing rate of temperature becomes more slowly with the increase in $\alpha_s/\alpha_l$. A good consistence can be seen between the simulation results and
analytical ones, which indicate the accuracy of the LBM model in dealing with the solid–liquid phase change problem.

**Case 2:** Fluid–solid conjugate heat transfer of dual-component materials with two basic structures

In this case, the fluid–solid conjugate heat transfer problems of dual-component materials (solid and fluid) with basic structure: parallel mode and series mode are considered with \( 100 \times 100 \) uniform lattice system. The schematics of this problem are shown in Figure 6. Assuming the thermal conductivity of each material is \( k_s \) for solid phase and \( k_f \) for liquid phase. The left and right walls are held at constant temperature \( T_h \) and \( T_c \). The top and bottom walls are adiabatic. The porosity (namely, the proportion of liquid phase) is \( \varepsilon \). The analytical solution of effective thermal conductivity for the basic structures can be deducted as:

\[
\begin{align*}
\text{Series mode} & \quad k_{\text{eff}} = \frac{1}{\varepsilon + \frac{1}{k_s}} \\
\text{Parallel mode} & \quad k_{\text{eff}} = \varepsilon k_f + (1 - \varepsilon) k_s
\end{align*}
\]  

\( (27) \)

To meet the requirement of temperature and the heat flux continuities at the phase interfaces, we have to assume identical volume thermal capacities \( (\rho C_p) \) for the fluid phase and solid phase; the fluid–solid conjugate heat transfer problem is thus solved and these assumptions will not affect the effective thermal conductivity. The temperature can be calculated using Eq. (15), and the heat flux can then be written as [23]

**Table 1. Comparisons between LBM and analytical solutions.**

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<th>Analytical solution</th>
<th>LBM solution</th>
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<td>0.276</td>
<td>0.456</td>
</tr>
<tr>
<td>0.6</td>
<td>2:1</td>
<td>0.313</td>
<td>0.313</td>
<td>0.178</td>
<td>0.350</td>
<td>0.351</td>
<td>0.195</td>
</tr>
<tr>
<td></td>
<td>5:1</td>
<td>0.147</td>
<td>0.147</td>
<td>0.042</td>
<td>0.260</td>
<td>0.261</td>
<td>0.340</td>
</tr>
<tr>
<td></td>
<td>10:1</td>
<td>0.078</td>
<td>0.077</td>
<td>0.830</td>
<td>0.230</td>
<td>0.231</td>
<td>0.389</td>
</tr>
</tbody>
</table>

LBM, lattice Boltzmann method.
\[ q = \sum_{i=0}^{3} (e_i g_i) \frac{\tau - 0.5}{\tau} \]  

After the temperature field is obtained, the effective thermal conductivity \( k_{\text{eff}} \) can be determined

\[ k_{\text{eff}} = \frac{l \cdot \int q dA}{\Delta T \int dA} \]

where \( q \) is the steady heat flux through the media cross-sectional area \( dA \) between the temperature difference \( \Delta T (\Delta T = T_h - T_c) \) with a distance \( l \).

Table 1 shows the calculated effective thermal conductivities compared with the analytical solutions of dual-component materials with series mode and parallel mode, when \( \varepsilon \) and \( k_s, k_l \) are different. We keep the dimensionless \( k_s \) as 0.5, and change \( k_l \) from 0.25 to 0.05. The range of porosity \( \varepsilon \) is set to 0.4–0.6. Through a lot of contrast, it can be found that our results agree very well with the analytical solutions, the maximum relative deviations of effective thermal conductivity are 1.018\% for series mode and 0.510\% for parallel mode. Therefore, our approach is effective for heat transfer problem in multiphase medium.

5. Results and discussion

In this section, we discuss the freezing process of saturated soil. The schematic of this problem is shown in Figure 7. The dimension of the physical model is 20 cm long and 8 cm wide. The initial temperature of soil \( T_i \) is 10°C uniformly, and the frozen temperature \( T_f \) is 0°C. The top face is cooled and kept at a constant temperature \(-10°C\) and the bottom and side faces are adiabatic. The saturated soil we selected is sandy loam soil and the thermophysical properties are given in Table 2.

In the lattice Boltzmann model, 500 \( \times \) 200 lattices are divided to maintain the same aspect ratio as the physical model. According to the references [6, 36], the porosity of sandy loam soil is

Figure 7. Schematic diagram of calculation model: (a) physical model, (b) lattice Boltzmann model. The dimension of the physical model is 20 cm long and 8 cm width. The initial temperature of soil \( T_i \) is 10°C uniformly, and the frozen temperature \( T_f \) is 0°C. The top face is cooled and kept at a constant temperature \(-10°C\) and the bottom and side faces of soil sample are adiabatic.
generally between 0.4 and 0.5. Thus, to study the influence of porosity on the freezing soil, 0.4, 0.45, and 0.5 are set in this work. The QSGS method is adopted to generate microstructure of soil. The initial distribution probability \( p_c \) is given as 0.35, the growth probability \( p_i \) is set as 0.01 in each direction. Figure 8 is the microstructure diagram when porosity \( \varepsilon \) is 0.45. The adiabatic boundary in physical model is set with nonequilibrium extrapolation approach. In the process of numerical simulation, the lattice unit is used in all variables. The relevant instructions of unit conversion are described in Section 4.3.

As mentioned previously, the evolution of temperature field during soil freezing process is influenced by water content, latent heat of ice-water phase change, different thermal conductivities of water and ice, and so on. In this section, considering the above effects, the development trend of temperature field is calculated in saturated soil with the porosity of 0.4, 0.45, and 0.5.

Figure 9 shows the temperature distribution in the section of \( x = 4 \) cm after 6, 12, 24, and 36 h of freezing. As the soil particles have larger thermal conductivity than water, the whole effective thermal conductivity would be larger with smaller porosity which results in a rapid temperature decrease at the beginning Figure 9(a). Before soil is completely frozen, a slope variation can be observed on frozen front with \( T = 0^\circ \)C. It is because when water changes into ice, the thermal conductivity would increase as much as 4 times larger than it before, thus temperature drops faster in completely frozen zone than in partly frozen zone Figure 9(b). This phenomenon becomes obvious in Figure 9(c) when the soil with porosity of 0.4 first reaches completely frozen state. Meanwhile, frozen processes are still slowly advanced in soil with porosity of 0.45 and 0.5. The mechanism is similar to the series mode of the fluid–solid conjugate heat conduction problem, which is shown in Figure 8(b). Due to the water blocking effect, the overall temperature decreases slowly before

<table>
<thead>
<tr>
<th></th>
<th>Thermal conductivity ( k/(W \cdot m^{-1} \cdot K^{-1}) )</th>
<th>Specific heat ( C_p/(J \cdot kg^{-1} \cdot K^{-1}) )</th>
<th>Density ( \rho/(kg \cdot m^{-3}) )</th>
<th>Latent heat ( L_f/(kJ \cdot kg^{-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandy loam soil</td>
<td>1.62</td>
<td>850</td>
<td>2215</td>
<td>—</td>
</tr>
<tr>
<td>Water</td>
<td>0.56</td>
<td>4180</td>
<td>1000</td>
<td>336</td>
</tr>
<tr>
<td>Ice</td>
<td>2.24</td>
<td>2050</td>
<td>917</td>
<td>336</td>
</tr>
</tbody>
</table>

Figure 8. The reconstructed soil microstructure (the porosity is 0.45) with QSGS method: black nodes denote interior (isolated) solids; white nodes are the pores. Note: QSGS, quartet structure generation set.
completely frozen. When the ice connected throughout the whole area, frozen speed would be greatly accelerated Figure 9(d).

Figure 10 shows the temperature development with time at the location of $y = 5, 10$, and $15$ cm. The initial freezing time of soil is about $14, 8$ and $2$ h at the location of $y = 5, 10$, and $15$ cm, respectively. It indicates that cold source on the top boundary produces a larger thermal gradient nearby, which could force soil to freeze quickly, and soil porosity has little influence on it.

In Figure 10(a), three frozen stages can be defined according to the slope. The first stage is non-frozen stage with temperature above $0^\circ C$. In this stage, all of the three soil samples with different porosity reach $0^\circ C$ almost at the same time. The second stage is partly frozen stage with temperature between $0^\circ C$ and $-2^\circ C$. During this stage, temperature decreases slowly as it spends another $12$ h to reach $-2^\circ C$ from $0^\circ C$. It is the process of breaking the water blocking effect. The third stage is completely frozen stage with temperature below $-2^\circ C$. Porosity impact gets evident in this stage. Even if the soil of $0.5$ porosity approaches fully frozen later than other two samples, it takes the least time to reach the final frozen temperature. It is due to the change of thermal conductivity when water freezes into ice. Therefore larger porosity would provide greater thermal conductivity after frozen. In Figure 10(b) and Figure 10(c), the three stages reduce into two with the second one disappeared. As explained previously, these two locations $y = 10$ and $15$ cm are close to the cold source, the higher thermal gradient is the dominant mechanism and makes the water blocking effect less important. Hence the porosity does not cause quite differences in the freezing process.

![Figure 9](image-url)  
**Figure 9.** Temperature distribution in the section of $x = 4$ cm after $6$, $12$, $24$, and $36$ h freezing, when the porosity $\varepsilon = 0.4$, $0.45$, and $0.5$ respectively.
6. Conclusion

To study the heat transfer with phase change in saturated soil during freezing process, we have developed a LBM model with double distribution functions to simulate temperature field evolution. This model has been validated by two examples: case 1 is to testify its accuracy in solid–liquid phase change of pure substance, and case 2 is to prove its effectiveness in dealing with conjugate heat transfer problem of dual-component materials with two basic structures: parallel mode and series mode. Both numerical solutions in the model have a good agreement with the analytical results. At the end, we apply it to simulate freezing process in saturated porous soil. The microstructure of saturated soil is generated by the QSGS method. Based on our present model, the temperature field evolution of saturated sandy loam soil with the porosity of 0.4, 0.45, and 0.5 has been calculated during freezing process. The results demonstrate that the freezing process is affected by the porosity and location, and can be divided into three stages. The porosity would be the predominant factor when the location is far from the cold source, otherwise, thermal gradient would be the principal factor. In the former situation, the mechanism of water blocking effect controls the whole freezing development. While in the latter situation, greater thermal gradient forces freezing process in a rather faster manner.

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References


